Grade 12   Paper 1   /150   09.09.2022

1.1.1. $x^2 + 4x - 21 = 0$

$(x-3)(x+7) = 0$

$\therefore x = 3$ or -7

2

1.1.2. $x(2x-7) = 3$

$2x^2 - 7x - 3 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{73}}{4}$$

$= 3{,}89$ or $-0{,}39$

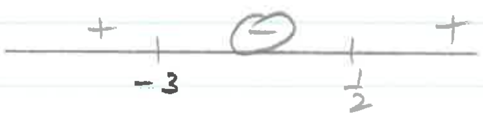
4

1.1.3. $(2x+3)(x+1) < 6$

$2x^2 + 5x + 3 < 6$

$2x^2 + 5x - 3 < 0$

$(2x-1)(x+3) < 0$



$-3 < x < \frac{1}{2}$ A

4

1.1.4. $2\sqrt{x} + x = 3$

$2\sqrt{x} = 3 - x$

$(2\sqrt{x})^2 = (3-x)^2$

$4x = 9 - 6x + x^2$

$0 = x^2 - 10x + 9$

$= (x-9)(x-1)$

$\therefore x = \cancel{9}$ or 1

reject

5

1.1.5. $2y + x + 3 = 0$

$x = -2y - 3$

$x^2 + y^2 + 2xy = 1$

$(-2y-3)^2 + y^2 + 2(-2y-3)y = 1$

$4y^2 + 12y + 9 + y^2 + 2(-2y^2 - 3y) = 1$

$5y^2 + 12y + 9 - 4y^2 - 6y - 1 = 0$

$y^2 + 6y + 8 = 0$

$(y+4)(y+2) = 0$

$y = -4$ or -2

$\therefore x = -2(-4) - 3$ or $-2(-2) - 3$

$= 5$ $= 1$

$\therefore x = 5$ and $y = -4$

or

$x = 1$ and $y = -2$

6

1.3 $k^{\frac{1}{x}} = 3$ $k^{\frac{1}{y}} = 4$

$k^{\frac{1}{w}} = 12$
$= 3 . 4$
$= k^{\frac{1}{x}} . k^{\frac{1}{y}}$ ✓
$= k^{\frac{1}{x}+\frac{1}{y}}$ ✓

$\therefore \frac{1}{w} = \frac{1}{x} + \frac{1}{y}$ ✓

LCD = wxy
($\because w \neq 0, x \neq 0, y \neq 0$)
$\times$ thru

$\frac{1}{w}.wxy = \frac{1}{x}.wxy + \frac{1}{y}.wxy$

$xy = wy + wx$
$= w(y+x)$ ✓

$\therefore \frac{xy}{y+x} = w$

$\therefore \frac{xy}{x+y} = w$

4

2.1. $d = 4$ $T_4 = 3x-1$
$T_7 = 2x+8$

2.1.1. $T_4 = 3x-1$
$a+3d = 3x-1$
$a+3(4) = 3x-1$ ✓
$a = 3x-13$

$T_7 = 2x+8$
$a+6d = 2x+8$
$a+6(4) = 2x+8$ ✓
$a = 2x-16$

$\therefore 3x-13 = 2x-16$
$x = -3$ ✓

3

2.1.2. (a) $a = 3x-13$ ✓
$= 3(-3)-13$ ✓
$= -22$ ✓

3

(b) $S_n = \frac{n}{2}(2a+(n-1)d)$ ✓

$S_{42} = \frac{42}{2}(2(-22)+41(4))$ ✓

$= 2520$ ✓

3

2.2. $61; \_; \_; \_; \ldots$

1^{st} diff : $T_k = 4k - 26$

2.2.1. 1^{st} diff

$T_1 = 4(1) - 26 = -22$

$T_2 = 4(2) - 26 = -18$

$61 ; 39 ; 21$

$-22 \quad -18$

2

2.2.2. $T_n = 2n^2 - 28n + 87$

min $T_n' = 0$

$4n - 28 = 0$

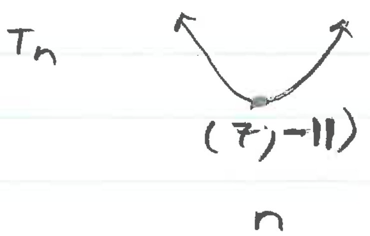
$n = 7$

$\therefore T_7 = 2(7)^2 - 28(7) + 87$

$= -11$

3

2.2.3.



$\therefore k > 11$

2

3.1. $p = 0,\dot{7} = 0,777\ldots$

3.1.1. $p = 0,7 + 0,07 + 0,007 + \ldots$

1

3.1.2. $a = 0,7 \quad r = \frac{1}{10}$ both

$T_n = ar^{n-1}$

$= 0,7\left(\frac{1}{10}\right)^{n-1}$

$\therefore p = \sum_{n=1}^{\infty} 0,7\left(\frac{1}{10}\right)^{n-1}$

3

3.1.3. $S_\infty = \frac{a}{1-r}$

$= \frac{0,7}{1 - 1/10}$

$= \frac{7}{9}$

2

3.2. $T_9 = ar^8 \quad T_{10} = ar^9 \quad T_8 = ar^7$

$T_9 + T_{10} = 6 . T_8$

$ar^8 + ar^9 = 6ar^7$

$\div a \qquad (a \neq 0)$

$r^8 + r^9 = 6r^7$

$\div r^7 \qquad (r \neq 0)$

$r + r^2 = 6$

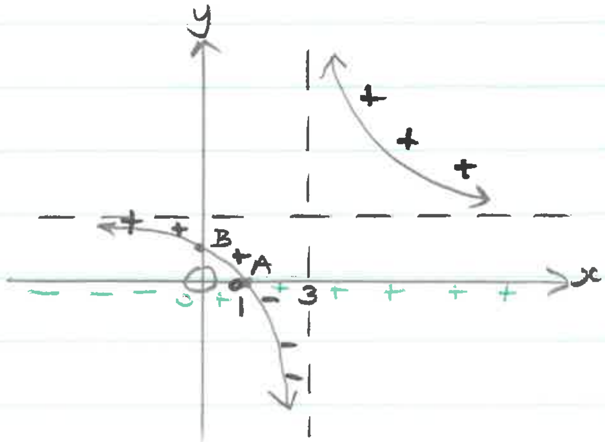
$r^2 + r - 6 = 0$

$(r-2)(r+3) = 0$

$\therefore r = 2$ or -3

4

4. $f: \quad y = \dfrac{x+k}{x+p}$



4.1. va: $x + p = 0$

$x = -p \qquad x = 3$

$\therefore \; -p = 3$

$p = -3$ ✓

(1)

4.2. $y = \dfrac{x+k}{x-3}$

sub $A(1;0)$

$0 = \dfrac{1+k}{1-3}$ ✓

$0 = \dfrac{1+k}{-2}$

$0 = 1+k$

$-1 = k$ ✓

(2)

$$\boxed{y = \frac{x-1}{x-3}}$$

4.3. y int: $x = 0$ $\quad y = \dfrac{0-1}{0-3} = \dfrac{1}{3}$

$\therefore \; B(0; \tfrac{1}{3})$ ✓ ✓value

(2)

4.4. $x \cdot f(x) \leq 0$

$x \quad y_f \quad \overline{0}$

$x \in (-\infty; 0]$ or $[1; 3)$

✓A ✓✓A

(3)

4.5.

$$\begin{array}{r} 1 \\ x-3 \overline{\smash{)}\, x - 1} \\ \ominus \quad \oplus \\ \underline{x - 3} \\ +2 \end{array}$$

✓

$\therefore \; y = 1 + \dfrac{2}{x-3}$

$\therefore \; f(x) = \dfrac{2}{x-3} + 1$ ✓

ie $f(x) = \dfrac{2}{x+(-3)} + 1$

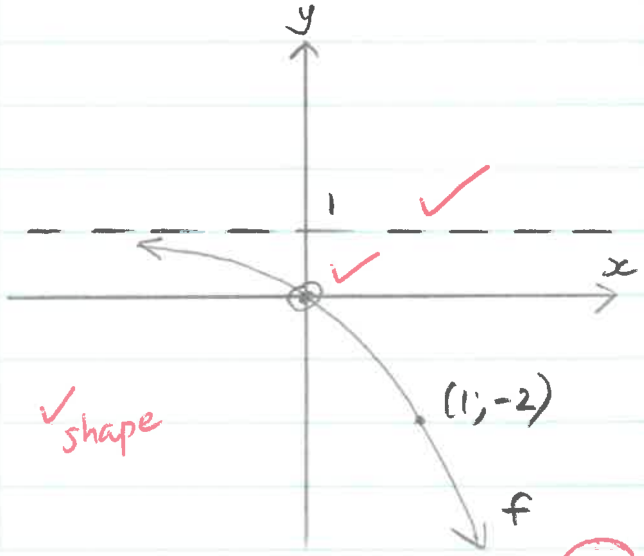
(2)

5. $f: \quad y = -3^x + 1$

$y = -1 \cdot 3^x + 1$

5.1. exponential

- $y$int: $y = -3^0 + 1$
  $= 0$
- $x$int $0 = -3^x + 1$
  $3^x = 1$
  $= 3^0$
  $x = 0$
- ha $y = 1$ ✓
- other pt: $x = 1 \quad y = -3^1 + 1$
  $= -2$
  $(1; -2)$



③

5.2 $y \in (-\infty; 1)$ ✓✓A

②

5.3. $f: \quad y = -3^x + 1$

$g: \quad g(x) = -f(x)$

$= -(-3^x + 1)$

$y = 3^x - 1$ ✓

$\therefore$ ha: $y = -1$ ✓

ans only 2/2

②

5.4. $g: \quad y = 3^x - 1$

$h: \quad y = 3^x$ ✓

$h^{-1}: \quad x = 3^y$ ✓

$y = \log_3 x$ ✓

③

6. $f: \quad y = x^2 - 4x - 11$

$g: \quad f'$

$y = 2x - 4$

$M(-1;t) \qquad N(7;10)$

6.1.1. D tp

$x = p$
$= \frac{-(-4)}{2(1)}$
$= 2$

$y = q$
$= (2)^2 - 4(2) - 11$
$= -15$

OR

$y' = 0$
$2x - 4 = 0$
$x = 2$

$D(2; -15)$ (3)

6.1.2. C x int f' ie g

$2x - 4 = 0$
$x = 2$

$C(2;0) \qquad N(7;10)$

$CN = \sqrt{(10-0)^2 + (7-2)^2}$
$= \sqrt{125}$
$= 11{,}18$ (4)

6.2.1. $f(x) < g(x)$

$y_f < y_g$

f under g

$x \in (-1; 7)$ (2)

[ ] 1/2

6.2.2. $g(x) - f(x)$
$= 2x - 4 - (x^2 - 4x - 11)$
$= 2x - 4 - x^2 + 4x + 11$
$= -x^2 + 6x + 7$

max : $x = p$
$= \frac{-(6)}{2(-1)}$
$= 3$ (4)

OR

max $-2x + 6 = 0$
$x = 3$

(6)

7.1. $A = P(1-i)^n$

$277\,63{,}12 = P\left(1 - \frac{17}{100}\right)^4$ ✓

$P = R\ 58\ 500$ ✓ (2)

7.2. $F = \frac{x[(1+i)^n - 1]}{i}$ — i, n ✓

$300\,000 = \frac{x\left[\left(1+\frac{8,6}{1200}\right)^{84} - 1\right]}{\frac{8,6}{1200}}$ ✓

$x = R\ 2\ 616{,}05$ ✓ (3)

7.3.1. $P = \frac{x[1-(1+i)^{-n}]}{i}$ — i, n ✓

$= \frac{8901{,}96\left[1-\left(1+\frac{10,4}{1200}\right)^{-300}\right]}{\frac{10,4}{1200}}$ ✓

$= 949\ 999{,}77\ldots$

$= R\ 950\ 000$ ✓ EN (3)

7.3.2. (a) OB_{204}

$P = \frac{8901{,}96\left[1-\left(1+\frac{10,4}{1200}\right)^{-96}\right]}{\frac{10,4}{1200}}$ ✓ — n ✓

$= R\ 578\ 551{,}24$ ✓ (3)

OR

A − F

$A = P(1+i)^n$

$= 950\ 000\left(1+\frac{10,4}{1200}\right)^{204}$

$= 5\ 523\ 928{,}83\ldots$

$F = \frac{8901{,}96\left[\left(1+\frac{10,4}{1200}\right)^{204} - 1\right]}{\frac{10,4}{1200}}$

$= 4\ 945\ 376{,}29\ldots$

A − F — ✓ n ✓ sub and both

$= R\ 578\ 552{,}54$ ✓

7.3.2. (b) $578\,551{,}24 = \frac{7500\left[1-\left(1+\frac{10,4}{1200}\right)^{-n}\right]}{\frac{10,4}{1200}}$ ✓

$\left(\frac{1513}{1500}\right)^{-n} = 0{,}331\ldots$ ✓

$-n = \frac{\log(0{,}331\ldots)}{\log\left(\frac{1513}{1500}\right)}$ ✓

$\therefore n = 127{,}96\ldots$

$\therefore$ 128 months ✓ (4)

OR $578\,552{,}54 = \frac{7500\left[1-\left(1+\frac{10,4}{1200}\right)^{-n}\right]}{\frac{10,4}{1200}}$

$\left(\frac{1513}{1500}\right)^{-n} = 0{,}331\ldots$

$-n = \frac{\log(0{,}331\ldots)}{\log\left(\frac{1513}{1500}\right)}$

$n = 127{,}96\ldots$

$\therefore$ 128 months

(7)

8.1. $f(x) = x - 3x^2$

$f(x+h) = x+h - 3(x+h)^2$

expa $= x+h - 3(x^2 + 2xh + h^2)$

$= x+h - 3x^2 - 6xh - 3h^2$

$f'(x)$

$= \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

sub $= \lim_{h \to 0} \dfrac{x+h-3x^2-6xh-3h^2-(x-3x^2)}{h}$

$= \lim_{h \to 0} \dfrac{x+h-3x^2-6xh-3h^2 - x + 3x^2}{h}$

$= \lim_{h \to 0} \dfrac{h - 6xh - 3h^2}{h}$ sim

$= \lim_{h \to 0} \dfrac{h(1 - 6x - 3h)}{h}$ cf

$= \lim_{h \to 0} (1 - 6x - 3h)$

$= 1 - 6x - 3(0)$

$= 1 - 6x$

5

8.2.1. $D_x \left[3x^4 - \frac{4}{x^2} \right]$

$= D_x \left[3x^4 - 4x^{-2} \right]$

$= 12x^3 + 8x^{-3}$

3

8.2.2. $y = a^2 x + 6\sqrt{x}$

$= a^2 x + 6x^{\frac{1}{2}}$

$\frac{dy}{dx} = a^2 + 3x^{-\frac{1}{2}}$

3

9. f: $y = x^3 - 3x + 2$

$y' = 3x^2 - 3 \quad y'' = 6x$

9.1. stat pt: $f' = 0$

f' ✓ $3x^2 - 3 = 0$ ✓o

$x^2 = 1$

$x = \pm\sqrt{1}$

$= \pm 1$

$x = -1: y = (-1)^3 - 3(-1) + 2$

$= 4$

✓ $(-1; 4)$

$x = 1: y = (1)^3 - 3(1) + 2$

$= 0$

✓ $(1; 0)$ 4

9.2. xint $0 = x^3 - 3x + 2$

From (9.1) $x = 1$ is on x-int

∴ $(x-1)$ is a factor

$x^3 + 0x^2 - 3x + 2$

$= (x-1)(x^2 + bx - 2)$

$-x^2 + bx^2 = 0x^2$

$bx^2 = 1x^2$

$b = 1$

✓ ∴ $(x-1)(x^2 + x - 2) = 0$

✓ $(x-1)(x+2)(x-1) = 0$

∴ $x = 1$ or -2 ✓ 3

∴ xints $(1;0)$ or $(-2;0)$

9.3.1. shape: $a = 1$

f decr

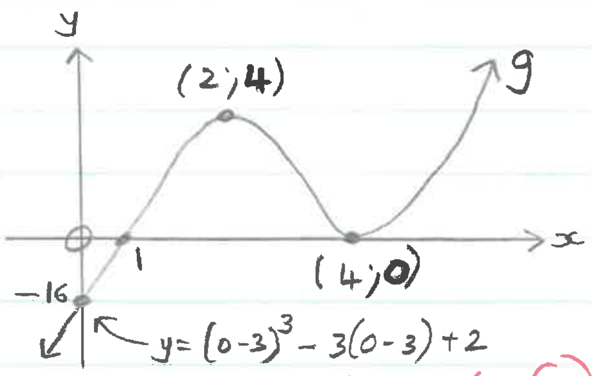
$x \in (-1; 1)$ ✓✓ 2

9.3.2. ∩ $f'' < 0$

$6x < 0$ ✓o

$x < 0$ ✓✓ 3

9.4. $x \rightarrow x - 3$ ∴ f 3 →



✓ xints ✓ yint ✓ tp's ✓ shape 4

9.5. $0 < k < 4$ ✓✓ A 2

10.1. $OA = -3x + 9$ ✓

$OC = x$ ✓ (2)

10.2 area OABC

$= A$

$= (x)(-3x+9)$

$= -3x^2 + 9x$ ✓

max $A' = 0$

$-6x + 9 = 0$ ✓

$x = \frac{3}{2}$

$\therefore y = -3(\frac{3}{2}) + 9$

$= \frac{9}{2}$

$\therefore B(\frac{3}{2}; \frac{9}{2})$ ✓ ✓ (4)

11.1.1. $x + 160 + 55 + 200 + 85 + 255 + 60 + 45 = 900$ ✓

$\therefore x = 40$ ✓ (2)

11.1.2. $P(\text{H only}) = \frac{200}{900}$

$= \frac{2}{9}$ ✓✓ (2)

11.1.3 % at least 2 sports

$= \frac{55 + 60 + 85 + 40}{900} \times 100$

$= \frac{240}{900} \times 100$

$= 26{,}67\,\%$ ✓✓ $\frac{80}{3}$ (2)

11.2 SPECTRUM (8)

11.2.1.(a) $8! = 40\,320$ ways ✓ (1)

11.2.1.(b) $\underline{2}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$

$\therefore 2 \times 7! = 10\,080$ ways ✓ ✓ (2)

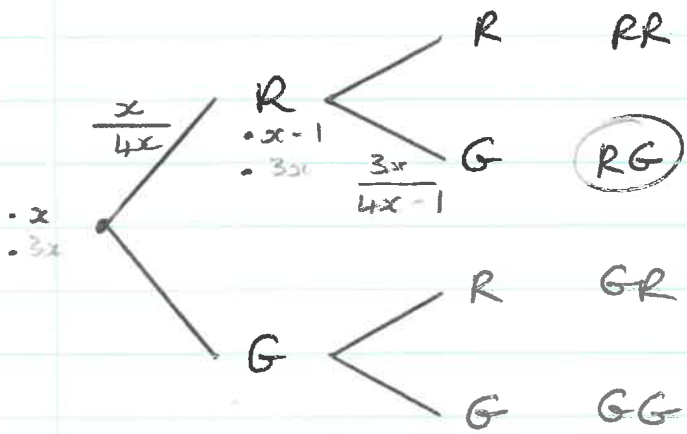
11.2.2. $\underbrace{TPR}_{3!}$ _ _ _ _ _ (6)

$P = \frac{6! \times 3!}{8!}$ ✓ ✓ 4320

$= \frac{3}{28}$ (2)

11.3. R : G

$1x \quad 3x$



$$P(RG) = \frac{1}{5}$$

$$\frac{1}{4} \times \frac{3x}{4x-1} = \frac{1}{5}$$

$$\times 4: \quad \frac{3x}{4x-1} = \frac{4}{5}$$

$$5(3x) = 4(4x-1)$$

$$15x = 16x - 4$$

$$4 = x$$

$\therefore$ balls in bag

$= 1x + 3x$

$= 4x$

$= 4(4)$

$= 16$ balls

5